Implementation of non-local operations for arbitrary high-dimensional systems with qubit quantum channel

Hao-Sheng Zeng*, Yong-Guang Shan, Jian-Jun Nie and Le-Man Kuang

Department of Physics, Hunan Normal University, Changsha 410081, People's Republic of China

Abstract

We propose a method to implement a kind of non-local operations between spatially separated two systems with arbitrary high-dimensions by using only low-dimensional qubit quantum channels and classical bit communications. The result may be generalized straightforwardly to apply for multiple systems, each of them with arbitrary dimensions. Compared with existed approaches, our method can economize classical resources and the needed low-dimensional quantum channels may be more easily established in practice. We also show the construction of the non-local quantum XOR gate for qutrit systems in terms of the obtained non-local operations as well as some single qutrit local gates.

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*E-mail adress: hszeng@hunnu.edu.cn

Entanglement is a very mystical and important phenomenon in quantum physics which has been studied extensively and deeply in theoretics and experiments. It has been playing a pivotal role in many aspects of quantum information and quantum computation, such as teleportation, dense coding , quantum cryptography and distributed quantum computation [1]. Recently, an interesting application of entanglement has been put forward which involves the implementation of non-local quantum gates or interactions [2–12] between spatially separated systems, by using a given resource of entangled states and applying local operations as well as classical communication. This subject is essentially important to realize the distributed quantum computation [13], because all quantum unitary operations between distributed systems can be realized by combining this non-local gate with local operations. The important aspect to implement the non-local gates is to increase the efficiency [7,11,12], given the limited resources including entangled qubits and classical bits, which is still worthwhile to be studied.

In addition, quantum entanglement in a high-dimensional Hilbert space have potential applications in quantum information and quantum computation [14,15]. Since particles in high-dimensional systems, i.e., qudits, can carry more information than qubits, it can increase information flux in quantum communication. It can also increase security [16] in quantum cryptography by exploiting the high-dimensional systems. Further, high-dimensional systems have advantages in fault-tolerant quantum computation and quantum error-correction [17]. Therefore, extending non-local operations to high-dimensional systems is also important. Reference [6] presented a method to implement a kind of non-local operations between multiple systems with arbitrary discrete dimensions by employing state-operator tool. However, it requires to consume high-dimensional entanglement resources, as well as employ classical-dit communication. In this paper, we show an approach for constructing a kind of non-local unitary operations between two spatially separated multi-level systems, by using only one low-dimensional quantum channel together with 2-cbit communication. Moreover, it can also be generalized straightforwardly to apply for multiple systems, each of them with arbitrary (maybe different) dimensions. The method has the advantages

of economizing classical resources and the needed low-dimensional quantum channels maybe more easily established in practice. In addition, we also discuss the applications of this kind of non-local operations in producing high-dimensional non-local entanglement states and performing distributed quantum computation.

Let us consider two-particle systems A and B which are remotely separated in space, the partners Alice and Bob previously share an accessorial entangled state

$$\left|\Psi_{abb_1}\right\rangle = \lambda_0 \left|0_a 0_b 0_{b_1}\right\rangle + \lambda_1 \left|0_a 0_b 1_{b_1}\right\rangle + \lambda_2 \left|1_a 1_b 0_{b_1}\right\rangle + \lambda_3 \left|1_a 1_b 1_{b_1}\right\rangle . \tag{1}$$

Here, the accessorial particle a belongs to Alice and b, b_1 to Bob. For simplicity, we assume that the coefficient's λ_i are non-negative real numbers and normalized as: $\sum_{i=0}^{3} |\lambda_i|^2 = 1$. We can easily get the entanglement of this state as

$$E(\left|\Psi_{abb_1}\right\rangle) = -H \log H - (1-H)\log(1-H). \tag{2}$$

Where $H = \lambda_0^2 + \lambda_1^2$ and the base of logarithm $\log(x)$ is taken to be 2.

In order to obtain the expected non-local operation on particles A and B, Alice and Bob firstly perform the following local unitary operations respectively [A kind of control-U operations with a, b_1 the control bits and A, B the target dits respectively.]

$$\begin{aligned} U_{aA} &= \left|0_a\right\rangle \left\langle 0_a\right| \otimes I_A + i \left|1_a\right\rangle \left\langle 1_a\right| \otimes U_A, \\ U_{b_1 B} &= \left|0_{b_1}\right\rangle \left\langle 0_{b_1}\right| \otimes I_B + \left|1_{b_1}\right\rangle \left\langle 1_{b_1}\right| \otimes U_B. \end{aligned} \tag{3}$$

Here U_{aA} ($U_{b_1 B}$) denotes that the unitary operation is performed between accessorial particle a (b_1) and target particle A (B). After these local unitary operations, they get a stator of the following form

$$S_1 = \lambda_0 \left|0_a 0_b 0_{b_1}\right\rangle + \lambda_1 \left|0_a 0_b 1_{b_1}\right\rangle \otimes U_B + i\lambda_2 \left|1_a 1_b 0_{b_1}\right\rangle \otimes U_A + i\lambda_3 \left|1_a 1_b 1_{b_1}\right\rangle \otimes U_A U_B. \tag{4}$$

Then Alice performs a measurement of σ_{xa} on stator S_1 with respect to the accessorial particle a, and transmits his measurement outcome (through one classical bit) to Bob. Following this message, Bob will do nothing or perform an operation of σ_{zb} on his accessorial

particle b, conditional on his received information 1 or -1. After this process, the stator becomes

$$S_2 = \lambda_0 \left|0_b 0_{b_1}\right\rangle + \lambda_1 \left|0_b 1_{b_1}\right\rangle \otimes U_B + i\lambda_2 \left|1_b 0_{b_1}\right\rangle \otimes U_A + i\lambda_3 \left|1_b 1_{b_1}\right\rangle \otimes U_A U_B \tag{5}$$

In order to eliminate accessorial particles b and b_1 and get the expected non-local operation, Bob now collectively measures his particles b and b_1 in the following Bell-like basis

$$\begin{aligned}
|B_1\rangle &= \cos\alpha|0_b 0_{b_1}\rangle + \sin\alpha|1_b 1_{b_1}\rangle, \\
|B_2\rangle &= \cos\alpha|1_b 1_{b_1}\rangle - \sin\alpha|0_b 0_{b_1}\rangle, \\
|B_3\rangle &= \cos\beta|0_b 1_{b_1}\rangle + \sin\beta|1_b 0_{b_1}\rangle, \\
|B_4\rangle &= \cos\beta|1_b 0_{b_1}\rangle - \sin\beta|0_b 1_{b_1}\rangle,
\end{aligned} \tag{6}$$

where α and β are real numbers. The corresponding probability that S_2 projected onto each basis is

$$\begin{aligned}
P_1 &= \lambda_0^2 \cos^2\alpha + \lambda_3^2 \sin^2\alpha, \\
P_2 &= \lambda_0^2 \sin^2\alpha + \lambda_3^2 \cos^2\alpha, \\
P_3 &= \lambda_1^2 \cos^2\beta + \lambda_2^2 \sin^2\beta, \\
P_4 &= \lambda_1^2 \sin^2\beta + \lambda_2^2 \cos^2\beta,
\end{aligned} \tag{7}$$

respectively. After getting one of the resulting operators, Bob sends 1-cbit information to inform Alice whether she will perform U_A (for outcomes P_2 and P_4) or not (for outcomes P_1 and P_3). At the same time, Bob will also perform a local operation of U_B (for outcomes P_2 and P_3) or do nothing (for outcomes P_1 and P_4). Then they can obtain respectively non-local unitary operators

$$\begin{aligned}
U_{1AB} &= P_1^{-1/2}[\lambda_0 \cos\alpha + i\lambda_3 \sin\alpha U_A U_B], \\
U_{2AB} &= P_2^{-1/2}[\lambda_3 \cos\alpha + i\lambda_0 \sin\alpha U_A U_B], \\
U_{3AB} &= P_3^{-1/2}[\lambda_1 \cos\beta + i\lambda_2 \sin\beta U_A U_B], \\
U_{4AB} &= P_4^{-1/2}[\lambda_2 \cos\beta + i\lambda_1 \sin\beta U_A U_B].
\end{aligned} \tag{8}$$

If we choose $\lambda_0 = \lambda_3$, $\lambda_1 = \lambda_2$ [Eq.(2) tells us that in this case, the entanglement of the quantum channel of eq.(1) is maximal.] and $\alpha = \beta = \xi$, then we get $U_{1AB} = U_{2AB} = U_{3AB} = U_{4AB}$ which have the following general form

$$U_{AB}(\xi) = \exp\left[i\xi U_A U_B\right]. \quad \xi \in [0, 2\pi]. \tag{9}$$

Thus, we obtain a non-local unitary operation between spatially separated systems A and B. The probability to produce this non-local operation is apparently one, and the angle ξ can be adjusted at will by Bob alone. Note that the local operators U_A and U_B for particles A and B are not only unitary, but also Hermitian.

It is worthwhile to point out that our approach to produce high-dimensional non-local operation can be regarded as a straightforward generalization of the method to produce non-local qubit operation [12]. Especially, for qubit systems, we can choose $U_A = \sigma_{nA}$ and $U_B = \sigma_{nB}$ with $n = x, y, z$, then eq.(9) reduces to the well known form of $\exp(i\xi\sigma_{nA}\sigma_{nB})$, which represents a general non-local operation in the sense that, along with single qubit local operations, it can realize any expected unitary operation between spatially separated qubits A and B.

Note also that, like the results presented by many authors [6,12], one of the prerequisites to determinately produce the non-local operation of eq.(9) is the utilizing of a maximally entangled quantum channel. However, imitating the processing offered by reference [12] and at the expense of successful probability, we can also get this non-local operation through the use of a lower entangled quantum channel.

Our result has extensive applications. It can be applied to the case of any high-dimensional systems, including A and B with different dimensions. In our method, we employ only low-dimensional qubit-entanglement resource. Thus compared with the approach [6] that using high-dimensional entanglement resources to produce the corresponding non-local operations, it is more simple and the needed entanglement resources are more easily obtained. Moreover, the classical consumptions in our scheme are always two cbits (i.e., bidirectional bit communication between Alice and Bob), rather than two classical dits [6].

Thus, it will economize many classical resources when higher dimensional systems involved.

The non-local operation of eq.(9) give us an appropriate room to realize a variety of interactions. In order to obtain a concrete non-local operation between two d-dimensional systems, it is needed for finding an adequate unitary as well as Hermitian operator U_d for systems A and B. If we regard the basis $\{|s\rangle\,;\ s=0,1,\ldots,d-1\}$ of a d-dimensional system as the eigenstates of angular momentum operator J_z of a spin system with eigenvalues $-j,\ldots,j$ respectively (j is the highest quantum number which is an integer or half-integer and satisfies $d=2j+1$), then a simple and intuitionistic choice for unitary and Hermitian operator U_d is

$$U_d = \begin{cases} \exp(i\pi J_n), & d \quad \text{odd} \\ i\exp(i\pi J_n), & d \quad \text{even} \end{cases}. \tag{10}$$

Of course, in a d-dimensional system, the choice for U_d is not single in general. There may exist many other forms of choices. As an example, we write out the general form of U_3 for a three-dimensional system, i.e., qutrit, in the basis $\{|0\rangle, |1\rangle, |2\rangle\}$

$$U_3 = \begin{bmatrix} a_1 & b_1 e^{i\varphi_1} & b_2 e^{i\varphi_2} \\ b_1 e^{-i\varphi_1} & a_2 & b_3 e^{i\varphi_3} \\ b_2 e^{-i\varphi_2} & b_3 e^{-i\varphi_3} & a_3 \end{bmatrix} \tag{11}$$

with parameters a_i, b_i and phases φ_i $(i=1,2,3)$ are all real. Through the investigation of the unitarity of U_3, we find that the phases φ_i only need satisfying one of the two conditions

$$\varphi_1 = \varphi_2 - \varphi_3, \tag{12}$$

$$\varphi_1 = \varphi_2 - \varphi_3 + \pi, \tag{13}$$

and the parameters a_i, b_i should satisfy the following set of equations

$$\begin{cases} a_1^2 + b_1^2 + b_2^2 = 1, \\ a_2^2 + b_1^2 + b_3^2 = 1, \\ a_3^2 + b_2^2 + b_3^2 = 1, \\ (a_1 + a_2)b_1 \pm b_2 b_3 = 0, \\ (a_1 + a_3)b_2 \pm b_1 b_2 = 0, \end{cases} \tag{14}$$

where the '+' in front of b_2b_3 and b_1b_2 corresponds to eq.(12), and '-' corresponds to eq.(13). Further investigation suggests that this set of equations has, and nay has, multiple sets of real number solutions. [Note that the number of variables are larger than that of equations.]

The non-local operation of eq.(9) presents a means of producing higher-dimensional non-local entanglement states in terms of lower-dimensional qubit-entanglement resources. As an example, let us show how to produce the 3-dimensional non-local maximally entangled state

$$\frac{1}{\sqrt{3}}[|0_A0_B\rangle + |1_A1_B\rangle + |2_A2_B\rangle]. \tag{15}$$

The process for the creation of this state requires the twice uses of eq.(9). Assume that the initial state of the bipartite system is $|0_A0_B\rangle$. First, we choose $U_i = |1_i\rangle\langle 0_i| + |0_i\rangle\langle 1_i| + |2_i\rangle\langle 2_i|$ with $i = A$ or B, and $\xi = \arcsin(\sqrt{2/3})$, then the action of eq.(9) on this initial state produces

$$\sqrt{\tfrac{1}{3}}\,|0_A0_B\rangle + i\sqrt{\tfrac{2}{3}}\,|1_A1_B\rangle\,. \tag{16}$$

Afterward, we let $U_A = |0_A\rangle\langle 0_A| + |1_A\rangle\langle 2_A| + |2_A\rangle\langle 1_A|$, $U_B = |0_B\rangle\langle 0_B| + i\,|1_B\rangle\langle 2_B| - i\,|2_B\rangle\langle 1_B|$ and $\xi = \pi/4$, it then produces

$$\frac{1}{\sqrt{3}}[e^{i\pi/4}\,|0_A0_B\rangle + i\,|1_A1_B\rangle + i\,|2_A2_B\rangle]. \tag{17}$$

Finally, an action of local operation of $e^{i\pi/4}\,|0_A\rangle\langle 0_A| + |1_A\rangle\langle 1_A| + |2_A\rangle\langle 2_A|$ will enable us to get the desired state of eq.(15). Note that the creation of this non-local 3-dimensional maximally entangled state involves the twice uses of eq.(9), which indicates the consumption of two maximally entangled qubit-resource states. This is consistent with the fact that local unitary operations and classical communication can not increase the entanglement [18]. Since the Von Neumann entropy is $\log 3$ for the state of eq.(15), and 1 for the entanglement resource used above, thus single utilizing of eq.(9) is not likely to produce the state of eq.(15).

The non-local operation of eq.(9) may also play an important role in multi-level distributed quantum computation and non-local quantum information processing. As an example, let us show that how to construct a XOR gate in terms of non-local operation of

eq.(9), along with single particle local operations. For simplicity, our discussion will be only limited to the case of 3-dimensional systems, i.e., qutrits. The XOR gate between qutrits A and B in this case is defined as

$$XOR_{AB}\left|j\right\rangle_A\left|k\right\rangle_B=\left|j\right\rangle_A\left|j\oplus k\right\rangle_B, \tag{18}$$

where the "$\oplus$" operation now indicates addition modulo 3. It has been pointed out [19] that the XOR operation can be decomposed into three operations

$$XOR_{AB}=F_AP_{AB}F_A^{-1} \tag{19}$$

where the Fourier transform for one qutrit is defined as

$$F\left|j\right\rangle=\frac{1}{\sqrt{3}}\sum_{l=0}^{2}e^{i2\pi jl/3}\left|l\right\rangle,\ j=0,1,2, \tag{20}$$

and the phase gate P_{AB} between qutrits A and B as

$$P_{AB}\left|j\right\rangle_A\left|k\right\rangle_B=\exp(i2\pi jk/3)\left|j\right\rangle_A\left|k\right\rangle_B. \tag{21}$$

Usually, we assume that the local unitary operations are easily obtained. Thus the task of finding a non-local XOR$_{AB}$ gate is equivalent to that of finding a non-local phase operation P_{AB} which, we will show below, can be constructed by the use of non-local operation of eq.(9) four times, along with some single qutrit operations. For clarity, we rewrite P_{AB} as

$$\begin{aligned}P_{AB}&=\left|00\right\rangle\left\langle 00\right|+\left|01\right\rangle\left\langle 01\right|+\left|02\right\rangle\left\langle 02\right|+\left|10\right\rangle\left\langle 10\right|+e^{i2\pi/3}\left|11\right\rangle\left\langle 11\right|\\&\quad+e^{i4\pi/3}\left|12\right\rangle\left\langle 12\right|+\left|20\right\rangle\left\langle 20\right|+e^{i4\pi/3}\left|21\right\rangle\left\langle 21\right|+e^{i2\pi/3}\left|22\right\rangle\left\langle 22\right|.\end{aligned} \tag{22}$$

Where we introduce abbreviation $\left|ij\right\rangle\equiv\left|i\right\rangle_A\left|j\right\rangle_B$ with $i,j=0,1,2$ to denote the basis of combined system. Now we first put $U_A=\left|0_A\right\rangle\left\langle 0_A\right|+\left|1_A\right\rangle\left\langle 1_A\right|-\left|2_A\right\rangle\left\langle 2_A\right|$, $U_B=\left|0_B\right\rangle\left\langle 0_B\right|-\left|1_B\right\rangle\left\langle 1_B\right|+\left|2_B\right\rangle\left\langle 2_B\right|$ and $\xi=\gamma$, then the non-local operation of eq.(9) becomes

$$\begin{aligned}U'_{AB}(\gamma)&=e^{i\gamma}\left|00\right\rangle\left\langle 00\right|+e^{-i\gamma}\left|01\right\rangle\left\langle 01\right|+e^{i\gamma}\left|02\right\rangle\left\langle 02\right|\\&\quad+e^{i\gamma}\left|10\right\rangle\left\langle 10\right|+e^{-i\gamma}\left|11\right\rangle\left\langle 11\right|+e^{i\gamma}\left|12\right\rangle\left\langle 12\right|\\&\quad+e^{-i\gamma}\left|20\right\rangle\left\langle 20\right|+e^{i\gamma}\left|21\right\rangle\left\langle 21\right|+e^{-i\gamma}\left|22\right\rangle\left\langle 22\right|,\end{aligned} \tag{23}$$

Afterward, we put $U_A = |0_A\rangle\langle 0_A| + |1_A\rangle\langle 1_A| - |2_A\rangle\langle 2_A|$, $U_B = |0_B\rangle\langle 0_B| + |1_B\rangle\langle 1_B| - |2_B\rangle\langle 2_B|$ and $\xi = \delta$, analogically we can get

$$\begin{aligned} U''_{AB}(\delta) = & e^{i\delta}|00\rangle\langle 00| + e^{i\delta}|01\rangle\langle 01| + e^{-i\delta}|02\rangle\langle 02| \\ & + e^{i\delta}|10\rangle\langle 10| + e^{i\delta}|11\rangle\langle 11| + e^{-i\delta}|12\rangle\langle 12| \\ & + e^{-i\delta}|20\rangle\langle 20| + e^{-i\delta}|21\rangle\langle 21| + e^{i\delta}|22\rangle\langle 22|. \end{aligned} \tag{24}$$

It is easily to check up that the non-local phase operation P_{AB} can be constructed as

$$P_{AB} = P_A P_B [U''_{AB}(\pi/6) S_A S_B U''_{AB}(\pi/6)][U'_{AB}(\pi/3) S_A S_B U'_{AB}(\pi/3)], \tag{25}$$

where $P_A = |0_A\rangle\langle 0_A| - |1_A\rangle\langle 1_A| - |2_A\rangle\langle 2_A|$, $P_B = -|0_B\rangle\langle 0_B| + |1_B\rangle\langle 1_B| + |2_B\rangle\langle 2_B|$ are the single qutrit phase gates, and $S_i = |0_i\rangle\langle 0_i| + |1_i\rangle\langle 2_i| + |2_i\rangle\langle 1_i|$ with $i = A$, B the single qutrit swap gates between levels $|1\rangle$ and $|2\rangle$. The non-local operations $U'_{AB}(\pi/3)$ and $U''_{AB}(\pi/6)$ are given by eqs.(23) and (24) with $\gamma = \pi/3$, $\delta = \pi/6$ respectively.

At the end of this paper, we point out that, combining the technique in [12] with ours, we can easily generalize the result of eq.(9) to the case of containing multiple systems and realize a non-local operation between multiple spatially separated particles, each of them with arbitrary (maybe different) dimensions. The concrete form for this non-local operation is

$$U_{A_1 A_2 \ldots A_N}(\xi) = \exp[i\xi U_{A_1} U_{A_2} \ldots U_{A_N}], \tag{26}$$

where A_i with $i = 1, 2, \ldots, N$ denote N spatially separated particles, and U_{A_i} the corresponding local operators that satisfied $U^{+}_{A_i} = U_{A_i}$ and $U^{+}_{A_i} U_{A_i} = U_{A_i} U^{+}_{A_i} = 1$. The consumptions for producing this multi-part and multi-dimensional non-local operation are $(N-1)$ pairs of classical bit communication and a non-local quantum entangled qubit resource between N partners, attached by some quantum local unitary operations and quantum measurements. However, for the method presented in reference [6], there will be needed N pairs of entangled qudit resource along with N pairs of classical dit communication.

In conclusion, we have proposed a method to implement a kind of non-local operations between spatially separated two systems with arbitrary high dimensions, by using

only low-dimensional qubit quantum channels and classical bit communications. The result may be generalized straightforwardly to apply for multiple systems, each of them with arbitrary dimensions. Compared with existed approaches, our method can economize many classical resources when higher-dimensional and multiple systems involved, and the needed low-dimensional quantum channels may be more easily established in practice. We have also shown the construction of the non-local quantum XOR gate for qutrit systems in terms of the obtained non-local operations as well as some single qutrit local gates, which suggests the universality of the obtained non-local operations in multi-level distributed quantum computation and non-local quantum information processing.

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REFERENCES

[1] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information*, (Cambridge University Press, Cambridge, England, 2004).

[2] J. Eisert, K. Jacobs, P. Papadopoulos and M. B. Plenio, Phys. Rev. A **62**, 052317 (2000).

[3] B. Kraus and J. I. Cirac, Phys. Rev. A **63**, 062309 (2001).

[4] D. Collins, N. Linden and S. Popescu, Phys. Rev. A **64**, 032302 (2001).

[5] S. F. Huelga, J. A. Vaccaro, A. Chefles and M. B. Plenio, Phys. Rev. A **63**, 042303 (2001); S. Huelga, M. B. Plenio and J. A. Vaccaro, Phys. Rev. A **65**, 042316 (2002).

[6] B. Reznik, Y. Aharonov and B. Groisman, Phys. Rev. A **65**, 032312 (2002).

[7] J. I. Cirac, W. Dür, B. Kraus and M. Lewenstein, Phys. Rev. Lett. **86**, 544 (2001); W. Dür, and J. I. Cirac, Phys. Rev. A **64**, 012317 (2001).

[8] W. Dür, G. Vidal, J. I. Cirac, N. Linden and S. Popescu, Phys. Rev. Lett. **87**, 137901 (2001).

[9] W. Dür, G.Vidal and J. I. Cirac, Phys. Rev. Lett. **89**, 057901 (2002).

[10] Y. F. Huang, X. F. Ren, Y. S. Zhang, L. M. Duan and G. C. Guo, Phys. Rev. Lett. **93**, 240501 (2004).

[11] B. Groisman and B. Reznik, Phys. Rev. A **71**, 032322 (2005).

[12] L. Chen and Y. X. Chen, quant-ph/0501107.

[13] J.I. Cirac, A.K.Ekert, S. F. Huelga and C. Macchiavello, Phys. Rev. A **59**, 4249 (1999).

[14] S. D. Bartlett, H. de Guise and B. C. Sanders, Phys. Rev. A **65**, 052316 (2002).

[15] R. T. Thew, K. Nemoto, A. G. White and W. J. Munro, Phys. Rev. A **66**, 012303 (2002).

[16] D. Bruss and C. Macchiavello, Phys. Rev. Lett. **88**, 127901 (2002); N. J. Cerf, M. Bourennane, A. Karlsson and N. Gisin, *ibid.* **88**, 127902 (2002).

[17] E. Knill, *Fault-tolerant postselected quantum computation: schemes,* quant-ph/0402171 (2004).

[18] V. Vedral, M. B. Plenio, M. A. Rippin and P. L. Knight, Phys. Rev. Lett. **78**, 2275 (1997).

[19] A. B. Klimov, R. Guzman, J. C. Retamal C. Saavedra, Phys. Rev. A **67**, 062313 (2003).